ON SURVO CROSS SUM PUZZLES

SEPPO MUSTONEN

Consider an $m \times n$ contingency table with missing cell frequencies. Thus only the marginal frequencies are available like in the following 3×4 case:

	1	2	3	4	
1	?	?	?	?	13
2	?	?	?	?	27
3	?	?	?	?	38
	13	14	23	28	78

The marginal frequencies tell practically nothing about the cell counts. In fact, in this case there are a huge amount (1104682, a good exercise in combinatorics!) of different tables having the same margins, two extreme items being

	1	2	3	4	
1	0	0	0	13	13
2	0	0	12	15	27
3	13	14	11	0	38
	13	14	23	28	78

	1	2	3	4	
1	13	0	0	0	13
2	0	14	13	0	27
3	0	0	10	28	38
	13	14	23	28	78

with correlation coefficients $r = -0.78$ and $r = 0.92$, respectively, when equally spaced scores are assumed.

The grand total in these tables is 78 which happens to be equal to $1+2+3+4+5+6+7+8+9+10+11+12$. Can then the integers $1, 2, \ldots, 12$ in some order be cell frequencies in this table? Yes, they can, but in one way only!

Thus the requirement that an $m \times n$ table with given marginal frequencies and a grand total equal to $mn(mn+1)/2$ has to filled by integers $1, 2, \ldots, mn$ so that the sums of rows and columns are equal to the given marginal values is very restrictive indeed and this is the task when solving **Survo (cross sum) puzzles**.

Statistical Days in Tampere 10-11 May 2007 seppo.mustonen@survo.fi

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A Survo puzzle with only marginal sums given and no cell frequencies available is called an **open** Survo puzzle.

On the other hand, it is easy to create, for example, 3×4 tables with 'cell counts' $1, 2, \ldots, 12$ in some order. There are altogether $12! = 479001600$ such tables but only 83952 (0.0175 per cent) of them presented as open Survo puzzles have a unique solution. Of these 83952 tables only $83952/(3! \cdot 4!) = 583$ are essentially different when tables obtained from each other by row and column permutations are considered similar.

For example, the open 3×3 Survo puzzle (displayed in standard notation with letters as column headers)

	A	B	C	
1				12
2				17
3				16
	14	15	16	

has 26 different solutions but if the number (C1=2) in the right upper corner is given,

	A	B	C	
1			2	12
2				17
3				16
	14	15	16	

the solution becomes unique.

From April 2006 I have presented Survo puzzles for various dimensions m, n and often (in order to guarantee uniqueness of the solution) with also some 'cell frequencies' given. Most of the problems have been published on the web page
`http://www.survo.fi/puzzles`
The Finnish version
`http://www.survo.fi/ristikot`
contains still more information.
The most comprehensive papers on the topic are
`http://www.survo.fi/papers/puzzles.pdf` (in English),
`http://www.survo.fi/papers/ristikot.pdf` (in Finnish).

The degree of difficulty (MD) in Survo puzzles may vary from easy (suitable e.g. for teaching of arithmetics and plain reasoning) like this one ($MD = 0$)

	A	B	C	D	
1		1			23
2	10		2		31
3		4		6	24
	21	12	22	23	

to very hard or almost impossible (without using a computer) like this 4×5 open Survo puzzle ($MD = 7000$)

	A	B	C	D	E	
1						24
2						41
3						58
4						87
	21	37	42	50	60	

Larger cases are really difficult to solve. For example, as far as I know, nobody knows the number of essentially different 5×5, uniquely solvable open Survo puzzles. For the 4×4 case, the corresponding number is 5327 according to enumeration made by *Petteri Kaski*.

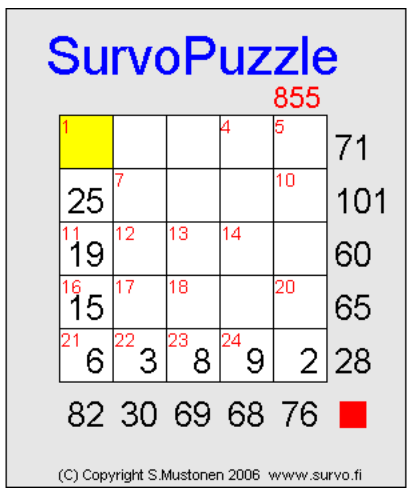
The measure of difficulty MD is based on the mean performance of the partially stochastic algorithm I programmed for finding the solution of any given Survo puzzle immediately after devising the idea of this type of a puzzle.

Although information has been distributed internationally about Survo puzzles from the very beginning, no signs of a systematic study related to similar problems elsewhere have been reported.

Most people are solving Survo puzzles by just trial and error and if one is lucky enough, the solution even for trickier problems may be found within an hour or two. It is usually much more demanding to solve a Survo puzzle so that at the same time one proves uniqueness of the solution. Until now, *Joonas Kauppinen*, *Olli Mustonen*, *Risto Nieminen*, *Reijo Sund*, and *Kimmo Vehkalahti* have presented good ideas for treating these problems.

Survo puzzles in the form of a quick game offer different challenges. The most advanced form of the quick game can be played on websites
`http://www.survo.fi/java/quick5x5.html` (in English),
`http://www.survo.fi/java/pika5x5.html` (in Finnish).

The task is to fill the 5×5 table by numbers $1, 2, \ldots, 25$ as quickly as possible so that the row and column sums of the numbers are equal to values given on the borders of the table. Always, at first, the player has to select a cell in the table where to put the next number. By clicking that cell it will be painted in yellow. Then the number the player wants to put in that cell is picked by clicking a cell containing that number in red. If a correct number is selected, it will be placed in the yellow cell and the score (displayed in red above the table) grows. If the choice is not correct, the score is decreased and an interval of two tones will be heard. This musical interval gives a clue for the right number and the player can improve the selection on this basis. The score is also reduced by one unit in a second. Since the main goal in this game is to collect a score as high as possible, the player must be quick, smart, and lucky.



Solving a Survo puzzle as a quick game

This text (with solutions for Survo puzzles and active URL links) is available in `http://www.survo.fi/papers/tampere07.pdf`.

Spoilers

Here are solutions of Survo puzzles in this paper by listing the numbers by rows:
Page 1 (3×4): 2 1 3 7 5 4 8 10 6 9 12 11
Page 2 (3×3): 4 6 2 7 1 9 3 8 5
Page 3 (4×5): 1 3 4 7 9 2 6 8 10 15 5 11 12 14 16 13 17 18 19 20